Number Sense Activities for College Science Classes

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Math 5064 Project

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EXECUTIVE SUMMARY

College science courses require students to use a variety of mathematical concepts and calculations, but science textbooks and lab manuals rarely include practice or review of number sense that would help students use and do math more effectively. It is generally assumed that students should already have those skills, and students who do not will struggle and fall behind. Math is sometimes considered to be the gatekeeper for science majors, preventing students from pursuing an interest in science because they have difficulty with math. In the biological sciences, the importance of quantitative reasoning has been emphasized recently in publications such as BIO2010: Transforming Undergraduate Biology Education for Future Researchers (NRC, 2003, <https://www.nap.edu/read/10497/chapter/1>), Vision & Change in Undergraduate Biology Education (AAAS, 2011, <http://visionandchange.org/files/2013/11/aaas-VISchange-web1113.pdf>), and Scientific Foundations for Future Physicians (AAMC-HHMI, 2009, <https://www.aamc.org/download/271072/data/scientificfoundationsforfuturephysicians.pdf>).

This set of lessons is designed around the set of objectives listed below, which are the most common issues students have in science classes I teach. The lessons can be used as a unit at the beginning of a science course, or each can be used as a stand-alone lesson to prepare for a particular topic that is coming up in a class or as review for a topic the students are having difficulties with during the class or lab. One difficulty in teaching these topics to college students is to do so in a way that is not condescending or that makes students feel that they are doing elementary school math. Many of the materials available for these topics are written for a younger audience, so these lessons have been selected and revised carefully to be appropriate for the college age student – they are fun but not juvenile and provide enough challenge to maintain interest.

Objectives:

1. Represent and compare quantities in length, area, volume, density, and rate
2. Represent numbers in different ways (a\*0.5 = a/2 = a\*(1/2); unit conversions)
3. Estimate & check work to see if answer makes sense
4. Consider accuracy, precision, and reproducibility, and identify sources of error
5. Understand place value, use decimal numbers and scientific notation correctly
6. Calculate percentages, proportions, and ratios and convert one form to another
7. Understand radicals and exponents and use them appropriately
8. Use correct order of operations and input equations into calculator properly
9. Organize data and information (create diagrams, tables, or other visual representations)
10. Create and interpret various types of graphs
11. Understand variables, and represent them with letters in equations
12. Improve problem solving and strategic thinking skills

Additional Sources:

Students can also practice some of these skills outside of class time using the website MathBench Biology Modules from the University of Maryland, at <http://mathbench.umd.edu/index.html>

Sample Assessment Questions:

1. A block of aluminum measures 3.52 x 2.67 x 2.55 cm and has a mass of 61.68 grams. Based on the chemical properties of aluminum, the density is expected to be 2.70 g/cm3.
	1. Calculate the density using the measurements provided.
	2. Calculate the % error for the calculated value of density.
	3. Is this aluminum block made of pure aluminum? Explain.
2. Two gears, A and B, are arranged so that the teeth of one gear mesh with the teeth of the other gear. Gear A turns clockwise and has 54 teeth. Gear B turns counterclockwise and has 36 teeth. If gear A makes 5.5 rotations, how many rotations will Gear B make?
3. At the zoo, there are two adult crocodiles named Clyde and Terry. When they arrived at the zoo, Clyde was 4 meters long and Terry was 5 meters long. Five years later, now fully grown, Clyde is 7 meters long and Terry is 8 meters long. Have Clyde and Terry grown the same amount? Explain.
4. The nonsteroidal medication naproxen has been prescribed for a patient, 1375 mg/day in divided doses. Each tablet contains 0.275 g. How many tablets should the patient take per day?
5. Create a data table based on this graph. What improvements would you make to this graph to better display the data?



1. The graph below shows enzyme activity as a function of incubation temperature. At what temperature(s) is the enzyme activity approximately half its maximum (the y-axis is linear)?



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LESSON 1: MEASUREMENTS

**Objectives:** 1, 2, 4, 5 **Time Required:** 2 days (or one 2-hour lab)

**Launch:** Measurement, particularly accurate and precise measurement, is a cornerstone of good science. Measurement can be done in a variety of units, depending on what is being measured, including distance, temperature, time, mass, area, volume, density, and rate. Within each category, there are multiple units that could be selected based on the situation (for example, volume could be measured in liters, milliliters, microliters, gallons, cups, or teaspoons). The precision of various tools should be discussed to introduce significant figures (that is, the markings on the tool are calibrated and known, but you can estimate another digit between the markings).

**Explore:** Students, working in groups, will be provided with a variety of measuring devices (rulers, meter sticks, calipers, graduated cylinders, beakers, pipets, balances, and thermometers) and asked to measure a variety of objects (such as table top, rectangular tray, water, salt, pennies, wood blocks, and metal cylinders). A checklist of items to measure and in what units can be put on the board or provided as a handout. Students will measure length, and use this to calculate area and volume. Students can also measure volume with beakers, graduated cylinders, and pipets, and compare accuracy and precision among the methods. A problem that can be introduced is to measure a volume of water, such as a full 120 mL beaker, with a graduated cylinder that only measures up to 100 mL, so that students have to figure out a procedure. Students will measure the dimensions of the metal cylinder to calculate volume, then use the volume displacement method in a graduated cylinder to measure volume and compare the accuracy of the two methods. Mass can be measured of the wood block and metal cylinder (both electronic balances and triple-beam balances should be used), then used with the volume of each to calculate density. The metal cylinder can be identified as a particular metal by looking up metal densities. Students will measure a mass of salt and mix it into a measured volume of water, then measure the mass of a specific volume of the salt water to calculate density. Students will be asked to convert between measurements in different units (e.g. meters or millimeters to centimeters, Fahrenheit to Celsius, milliliters to liters) to practice conversions and decimals.

**Share:** At various points during the activity, students can share strategies for measuring a particular object (e.g. which measuring tool was selected, how the tool was used, how any calculations were performed, and the accuracy of their method). Calculations for percent error and unit conversions should be shared and explained by each group.

**Summarize:** The instructor can lead a discussion on accuracy and precision of measuring tools and methods and emphasize the importance of using the correct tool in the correct way. Student abilities and results from this lab may indicate to the instructor what other lessons from this project will be needed for the students in the class (such as decimals, percentages, or others).

**Sources:** There are many varieties of this lab available online.

LESSON 2: PROPORTIONS, PERCENTAGES, AND RATIOS

**Objectives:** 6, 2, 9 **Time Required:** 1-2 days

**Launch:** Proportions, percentages, and ratios are used extensively in science. The instructor can show examples such as DaVinci’s Vitruvian man, how proportions are used to judge dog breeds, scales on a pine cone from base to tip, and several others. Being able to manipulate proportions and convert from ratios to fractions to percentages is a skill that is important for students to develop for use in future science labs. The instructor can begin the activity by asking students to write a procedure for converting a fraction (such as 3/8) into a percentage.

**Explore:** In Part 1, each pair of students receives one bag of Skittles. Students are asked to determine the proportion of each color in the bag and the percentage of each color. Then, they will determine the ratio of one color to another color. Questions can be asked about the Skittles, such as what proportion are either yellow or green, what proportion are not purple, how many red ones would there be if you had 15 bags? Students should explain the relationship between a proportion and a percent, and the relationship between a ratio and a percent. Students should also be asked to explain the relationship between a proportion and a ratio (e.g. compare the proportion of red Skittles in the bag to the ratio of red:total or red:not red). In Part 2, students will explore the Golden Ratio (phi) by measuring various distances of the human body (see following pages). Once they have determined the ratio phi, they will be given a picture of another organism (such as shark, fish, snail, sunflower, pinecone, bird, tiger’s face, ant, butterfly, etc.) and asked to find the golden ratio in the proportions of that organism.

**Share:** During and after Part 1, students will be asked to show how they converted from a proportion to a percentage (there may be different methods). Students can share ideas about how the three representations (proportion, percentage, ratio) are similar and different. During Part 2, students should compare the results for the ratios they are calculating (they should all obtain similar but not exactly the same ratio, due to variation in individuals).

**Summarize:** The instructor will review the forms of proportions, ratios, and percentages and offer any tips for conversion from one to another that have not already been suggested by students.

**Sources:**

“Are You a Golden Person?” Maths Magic, <http://www.consef.org/wp-content/uploads/2014/01/Are-you-a-golden-person-STEMCON.pdf>

Students can get more practice on these topics at: <http://www.open.edu/openlearn/science-maths-technology/mathematics-and-statistics/mathematics-education/ratio-proportion-and-percentages/content-section-0?intro=1>



Supplement: Can you find any other body dimensions that occur in a golden ratio?



Supplement: Can you find any other facial dimensions that occur in a golden ratio?



LESSON 3: DOES MY ANSWER MAKE SENSE?

**Objectives:** 3, 2, 5 **Time Required:** <1 day

**Launch:** Instructor plays a video clip of a mascot (Bucky the Badger) doing push-ups during a football game (<http://mrmeyer.com/threeacts/buckythebadger/>) and asks students to estimate how many push-ups Bucky would have to do during the entire game.

**Explore:** Students need to determine what information they need to know to make a good estimate of the total number of push-ups, and can ask the instructor questions to gather that information. Ideas and estimates will be shared and discussed. Other questions will be asked (Task 1, see following page), and then students will move on to estimating answers for math problems, no calculators allowed (Task 2, also following). Students should be prepared to share their estimation strategy in words (not just a number).

**Share:** During the activity, students will share strategies for estimating.

**Summarize:** The instructor will point out any strategies that have not been suggested by students, and provide feedback on those that have been suggested (for example, if a strategy would work in all cases or not).

**Sources:**

“Numbers and Estimation,” Lesson 1 of Unit 1 (p. 8-14) in*Transitioning to College and Careers,* SREB Readiness Courses: Texas Edition. <https://www.tasanet.org/cms/lib/TX01923126/Centricity/Domain/249/math-ready.pdf>

“Decimals – Is the Answer Reasonable?” *Building Strengths with Numeracy*, VALBEC. <http://www.valbec.org.au/building-strength-with-numeracy/docs/group-2-samples/Is-the-answer-reasonable.pdf>





Check the accuracy of these results by using estimation:

1. 27.9 \* 3.24 = 90.396
2. 22.168 \* 5.73 = 96.264
3. 684 \* 53 = 362,520
4. 4981 / 36 = 13.8361
5. 3.785 \* 91.6 = 346.706
6. 981.7 / 2.89 = 308.55
7. 3.51 + 28.4 + 51.3 + 194.7 = 723.17
8. 49.2 + 81.6 + 427.1 + 121.3 = 679.2
9. 36.25 \* 2 \* 4.17 \* 3 = 906.975
10. 4(3.15 + 9.814) = 518.56

LESSON 4: PARTS OF A SQUARE

**Objectives:** 7, 1, 2, 3, 6 **Time Required:**  1 day

**Launch:** Ask the question, “In what occupations or situations would you need to find the longest side of a right triangle if you knew the other two sides?” After some discussion, the instructor can show examples of how square roots and exponents are used in science.

**Explore:** Pairs of students will be given diagrams that show a square divided into smaller units of squares and triangles (see examples on following page). After defining a length or area as a unit of 1, students will need to determine the side lengths and areas of all the shapes inside the square. No calculators will be used.

**Share:** Each pair of students will share their diagram and solutions, explaining the strategies they used to go from the one defined unit to all other calculations.

**Summarize:** The instructor will review radicals and exponents, and can point out other skills the students practiced during this activity, such as ratios, proportions, fractions, estimation, and perhaps decimals.



 



LESSON 5: DECIMAL NUMBERS & SCIENTIFIC NOTATION

**Objectives:** 5 **Time Required:** 1 day

**Launch:** Since students are familiar with shorthand used in texting and Twitter, write on the board (or read aloud) a paragraph (such as the example on the following page) and ask them to convert it into a form they would send as a text or Tweet.

**Explore:** Show some large and small numbers (see examples on following page) and have students read them to each other and try to find a way to write them in a shorter format. The instructor might suggest trying to write each number using multiples or powers (e.g. what would happen if you divided the number by 1000 some number of times, or added 100 some number of times, etc.). After students have some solutions, they can be shared and scientific notation will be introduced. Next, students will practice ordering decimal numbers by playing the Spiraling Decimals game (see game board on following page) in which students take turns plotting a decimal value on a spiral number line from 0 to 1, and the first student to have three points in a row wins. If it is apparent that students in the class need additional work with decimal numbers, use a deck of cards for which the numbers 1-9 are Ace-9 and all other cards represent the value 0. Students draw three cards and make the smallest number possible with those three values. For example, if the student draws the cards 5, King, and 3, they should write it first in fractions (“0” \* 1/10, “3” \* 1/100, “5” \* 1/1000) and then in decimal format (.035). Students could also practice scientific notation here.

**Share:** Students can share their ideas for how to write the numbers in a shorter format. Students who do not know scientific notation should be selected by the instructor to go first, so that a few different solutions can be shown. Then a student who is familiar with scientific notation could be selected to show how that works. Strategies for the Spiraling Decimals game should be shared once each pair of students has played one or two rounds, then allow students to play again using a different strategy.

**Summarize:** The instructor will show several examples of scientific notation in scientific publications (journal articles, websites, etc.). Stress that the number to the left of the decimal place must be between 1 and 10. Have students multiply two large numbers in their calculator such that the product will be in scientific notation, and be sure students know what to look for (E). Also emphasize the importance of place value and 0’s by showing an example of how a mistake of writing 0.1 instead of 0.01 can make a huge difference (such as in the calculation of water level in a river in feet to the volume of water in cubic feet per second).

**Sources:**

“Scientific Notation classroom activity.” Utah Education Network, MathShorts, “Scientific Notation classroom activity.” <http://www.uen.org/mathshorts/downloads/scientific_notation_classroom_activity.pdf>

“Spiralling Decimals.” NRICH Enriching Mathematics. <https://nrich.maths.org/10326>





**Spiraling Decimals Game**

You need a partner, [a copy of the game board](https://nrich.maths.org/content/id/10326/SpirallingDecimals.pdf), and two different coloured pencils. Decide who goes first. Take turns to choose a number from the grid and mark it on the spiral. Keep taking turns until one of you has marked three numbers next to each other.

|  |  |  |  |
| --- | --- | --- | --- |
| 0.5 | 0.25 | 0.75 | 0.3 |
| 0.35 | 0.9 | 0.99 | 0.999 |
| 0.1 | 0.01 | 0.05 | 0.79 |
| 0.64 | 0.32 | 0.54 | 0.865 |


Can you work out a winning strategy?
Does it matter who goes first?
Does it matter which number you choose first?
Can you make up a different set of numbers which would make the game more challenging?

LESSON 6: CALCULATORS AND ORDER OF OPERATIONS

**Objectives:** 8, 3, 7 **Time Required:** <1 day

**Launch:** The instructor will write a complex series of operations on the board, such as

-92 + √25 – 4(12 ÷ 2 + 3) \* 7

and ask students to solve it with their calculators and write their answer on the board. It is likely that students will obtain several different answers (if not, skip this activity).

**Explore:** Ask students to work in pairs to recall the order of operations, and then compare with others until the class agrees on 1) parentheses, 2) roots and exponents, 3) multiply and divide, and 4) add and subtract. Then students can reevaluate the original problem and determine the best way to enter it into their calculator to obtain the correct answer. Other problems can be written on the board for students to try (see following page), using the strategies they have developed.

**Share:** Students work together to identify the order of operations, and share strategies for entry of a complex series of operations into their calculator. Students may demonstrate several successful methods and each can choose the one that is easiest for them to remember.

**Summarize:** The instructor will emphasize the order of operations and show any tricks for calculator use that have not been suggested by students.

**Sources:**

<http://cms.cerritos.edu/uploads/sc/DLAs/M005.1_Order_of_Operations_Using_a_Scientific_Calculator.pdf>











LESSON 7: ORGANIZING DATA

**Objectives:** 9, 10, 11 **Time Required:** 1 day

**Launch:** The instructor will review independent and dependent variables, then show various types of graphs and ask the class what kind of graph it is and when they would use it, as well as have students identify the variables represented in each graph. Then data tables will be shown and students will determine which type of graph would be most appropriate, as well as the independent and dependent variables.

**Explore:** Students will be given two scenarios (see below) for which they need to identify the independent and dependent variables and create a data table that could be used to collect data during the experiment. Then students will generate some probable data for the experiment and create a graph. Next, students will be given four graphs (see following) and create a data table for the data represented in each graph.

**Share:** Students will share their data tables and graphs, and discuss any differences among the groups.

**Summarize:** The instructor will review student contributions to the discussion and clarify any remaining issues regarding the relationship between variables, data tables, and graphs.

**Working with Experimental Data**

For each of these scenarios, identify the independent and dependent variables, create a data table that could be used for data collection, generate probable data that would result for this experiment, and construct an appropriate graph with a title and labeled axes.

*A botanist wants to know the effect of different colors of light on the growth of plants. He thinks that plants can survive best in white light. He buys 5 ferns of the same species, which are all approximately the same age and height. The researcher places one in white light, one in blue light, one in green light, one in red light and one in the closet. All of the ferns are planted in Miracle-Gro® soil and given 20 mL of water once a day for 2 weeks. After the two weeks, he observes the plants and makes measurements.*

*A fisheries biologist knows that the percent of fish eggs that hatch is affected by the temperature of the water in an aquarium. She is attempting to identify which water temperature will cause the highest percentage of fish eggs to hatch. The researcher sets up 5 aquariums at the following temperatures: 10°C, 20°C, 30°C, 40°C, and 50°C. She adds 50 fish eggs to each aquarium and records the number of eggs that hatch in each aquarium.*

Create data tables from these graphs, and provide a short description of the experimental design used:









LESSON 8: DILUTION SERIES

**Objectives:** 6, 12, 2, 1 **Time Required:** 1-2 days (or one 2-hour lab)

**Launch:** When a new disinfectant is invented, microbiologists need to determine the most effective concentration to use the disinfectant. It has to be a strong enough concentration to eliminate the bacteria, but dilute enough to be cost effective and not harmful to surfaces.

**Explore:** The most concentrated form of the new disinfectant is 100000 µg/L. You have 100 mL of this concentrated disinfectant in Beaker A (note: use a food coloring solution). Use a pipet to transfer 50 mL of the disinfectant into Beaker B, and add 50 mL water to Beaker B and stir. Then transfer 50 mL from Beaker B into Beaker C, add 50 mL water to Beaker C and stir. Then transfer 50 mL from Beaker C into Beaker D, add 50 mL water to Beaker D and stir. Finally, transfer 50 mL from Beaker D into Beaker E, add 50 mL water to Beaker E and stir. Label 5 cuvettes near the top with the letters A through E, and use a small pipet to fill each cuvette from the corresponding beaker. Zero the colorimeter as demonstrated, then use the colorimeter to determine the absorbance value for each solution. Absorbance is the amount of light that is blocked from passing through the solution, or scattered by the molecules in the solution. The higher concentration will let less light through, and more dilute concentrations will let more light through. Create a data table and record your results, then make a graph with a title and labeled axes.

What is the concentration of the disinfectant in each of the beakers? What is the absorbance value for each of the solutions? How do the concentration and absorbance values relate to each other?

Challenge problem: Create a dilution series that results in a concentration in Beaker E of 160 µg/L.

**Share:** Students can present their dilution series for the challenge problem and compare the procedures they developed to do the calculations.

**Summarize:** The instructor will review ratios and percentages in the context of this biological application and help students develop a generalized procedure for creating a dilution series.

**Sources:**

A simulation of this activity (without the colorimeter measurements of absorbance) is available from NRICH Enriching Mathematics, <http://nrich.maths.org/6164>

LESSON 9: STRATEGIC THINKING

**Objectives:** 12 **Time Required:** 1 day

**Launch:** Today we are going to play some games to develop your strategic thinking skills!

**Explore:** Game 1: Hit the Target. Play this game with a partner. Starting at 0, each player can add 1, 2, 3, or 4 points on their turn, and the player that hits the target of 23 points wins. For example, Player 1 adds 3 points (total = 3), Player 2 adds 2 points (total = 5), and so on until one player hits 23 and the game is over. Game 2: Nim. Stack a handful of counters into any number of piles. Players take turns removing 1 or more counters from a single pile. The person who takes the last counter wins. Game 3: Three Mens Morris. Each player gets three counters. Using the game board below, take turns placing one counter at a time on any of the circles. Then play continues as players take turns moving their counters along the lines to an empty circle, one counter moves one space per turn. The first person to get all three of their counters in a row wins.



**Share:** Play each game several times. What strategies are you using? Can you develop a strategy to win every time? Does it matter who starts the game? After each game has been played a few times, strategies will be shared with the class, then students can choose alternate strategies to test them and discuss which work better than others.

**Summarize:** The instructor should emphasize the importance of strategic thinking in scientific discovery, as well as just about any career.

**Sources:** Hit the Target: <https://nrich.maths.org/397>, Nim: <http://nrich.maths.org/402>, Three Men’s Morris: <http://nrich.maths.org/1183>

LESSON 10: BEER’S LAW: VARIABLES & RELATIONSHIPS

**Objectives:** 11, 9, 10, 6 **Time Required:** 2 days, or one 2-hour lab

**Launch:** Chemical laboratory analysis of blood and urine to determine the presence and concentration of alcohol has been employed for many years. These procedures do, however, require qualified medical personnel and raise many difficult questions regarding the violation of Fourth and Fifth Amendment rights. The sampling of breath is much less objectionable legally, although similar questions regarding a violation of rights have been raised. Most states have “implied consent” laws designed to deal with these objections.

The first breath alcohol instrument was the Harger “Drunkometer” (1937). Not until the Breathalyzer was invented in 1954 did law enforcement officers have a reliable aid in determining the blood alcohol content (BAC) of an individual suspected of driving under the influence of alcohol. With this test, a correlation can be made between the percentage of alcohol in breath and in the blood. In most states, a person driving a motor vehicle with a BAC of 0.08% or greater is considered to be driving under the influence of alcohol.

A direct determination of BAC can be made by reacting the alcohol in the blood with potassium dichromate. The percentage of alcohol is calculated by measuring the amount of potassium dichromate that is “used up” in the reaction. Potassium dichromate is a yellow chemical that, when reacted with alcohol, loses its color. The more alcohol in the blood, the paler the solution will be as less potassium dichromate will remain in the solution. An instrument called a colorimeter can detect small changes in the color of a solution by measuring the amount of light of a particular wavelength can pass through the solution. Lighter solutions allow more light to pass, while darker solutions block light.

We can measure the absorbance of solutions of known BAC with the colorimeter, make a graph of the known data, and draw a “best fit” line through the known points. Once the pattern is determined, the absorbance of an unknown solution can be plotted along the “best fit” line to determine its concentration (BAC). This is an application known as “Beer’s Law.”

**Explore:** Students will be provided with a colorimeter and a stock solution of 0.20 BAC (created by the instructor with food coloring solution), pipets for measuring volumes, and three “unknowns” to test from fictional DWI arrests, and will need to determine if any of the “unknowns” is over the legal limit in the state of MN. In order to do this, students will need to devise a method of creating other known concentrations of BAC, record data on absorbance of these solutions, create a graph, determine the y = a + bx equation of the best fit line, then plot the absorbances of the unknowns on the graph and use the equation for the best fit line to determine the BAC of each.

**Share:** Once student groups have been working for a while on their methodology, ideas can be shared for what concentrations of BAC are needed to be made from the 0.20 stock solution and how those should be produced. Graphs of known concentrations can be compared as another checkpoint to make sure each group is on track. Groups should be reminded that each colorimeter’s calibration may be slightly different, so graphs, equations, and results will not be exactly the same.

**Summarize:** The instructor will review independent and dependent variables, relationships between continuous variables, best fit line, and solving equations.

**Sources:** Adapted from Vernier software colorimeter experiment “Determining Concentration of a Solution: Beer’s Law” <https://www.vernier.com/experiments/chem-a/17/determining_the_concentration_of_a_solution_beers_law/>

LESSON 11: PROBLEM SOLVING STRATEGIES

**Objectives:** 12, 9, 11, 3 **Time Required:** 1 day

**Launch:** The instructor will ask the class, “What do you do when you are having trouble solving a math problem?” and write ideas on the board.

**Explore:** Students will be given a set of word problems and a list of strategies (see following page) and demonstrate each of the problem solving methods for each problem.

**Share:** Groups will take turns explaining how each strategy worked out for the problems, and discuss which strategies worked well all the time, or under what conditions certain strategies worked better than others.

**Summarize:** The instructor will ask students to write a reflection paper on problem solving strategies and what they learned from this experience. When students are stuck on problems in the future, the instructor can ask students which of the strategies from this activity they have tried, or remind them of other strategies they could try.

**Sources:**

New Zealand Maths, “Problem Solving Strategies.” <https://nzmaths.co.nz/problem-solving-strategies>

Problems from: <http://people.stfx.ca/rtmacdon/mathproblems/>

**Common Problem Solving Strategies**

1. Guess (this includes guess and check, guess and improve)
2. Act It Out (act it out and use equipment)
3. Draw (this includes drawing pictures and diagrams)
4. Make a List (this includes making a table)
5. Think (this includes using skills you know already)

Being Systematic, Keeping Track, Looking For Patterns and Using Symmetry are different from the strategies we have talked about above in that they are over-arching strategies. In all problem solving, and indeed in all mathematics, you need to keep these strategies in mind.

**Being systematic** may mean making a table or an organised list but it can also mean keeping your working in some order so that it is easy to follow when you have to go back over it. It means that you should work logically as you go along and make sure you don’t miss any steps in an argument. And it also means following an idea for a while to see where it leads, rather than jumping about all over the place chasing lots of possible ideas.

It is very important to **keep track** of your work. We have seen several groups of children acting out a problem and having trouble at the end simply because they had not kept track of what they were doing. So keeping track is particularly important with Act it Out and Using Equipment. But it is important in many other situations too. Children have to know where they have been and where they are going or they will get hopelessly muddled. This begins to be more significant as the problems get more difficult and involve more and more steps.

In many ways **looking for patterns** is what mathematics is all about. We want to know how things are connected and how things work and this is made easier if we can find patterns. Patterns make things easier because they tell us how a group of objects acts in the same way. Once we see a pattern we have much more control over what we are doing.

**Using symmetry** helps us to reduce the difficulty level of a problem. Playing Noughts and crosses, for instance, you will have realised that there are three and not nine ways to put the first symbol down. This immediately reduces the number of possibilities for the game and makes it easier to analyse. This sort of argument comes up all the time and should be grabbed with glee when you see it.

Finally **working backwards** is a standard strategy that only seems to have restricted use. However, it’s a powerful tool when it can be used. In the kind of problems we will be using in this web-site, it will be most often of value when we are looking at games. It frequently turns out to be worth looking at what happens at the end of a game and then work backward to the beginning, in order to see what moves are best.

**Problems**:

A farmer has 160m of fence to enclose a four sided field. He is going to use all or part of one side of his barn so he can make his field as big as possible. If his barn is 100 m long, find the maximum area of his field.

In the farmyard there are some pigs and some chickens. In fact there are 87 animals and 266 legs. How many pigs are there in the farmyard?

Find and solve a system of equations given the following information. Three people go to a store where there is a sale. Every shirt in the store are x dollars, all pants are y dollars and all sweaters are z dollars. One person buys two shirts, one pair of pants and one sweater, The second person buys one shirt, two pairs of pants and two sweaters. The third person buys three shirts and four pairs of pants. Find the price of each item if the first person spent $155 and the second and third person spent $235 each.

Alex, Fred and Thomas run at constant rates. In a race of 1,000m, Alex finished 200m ahead of Fred and 400m ahead of Thomas. When Fred finished, how far was he ahead of Thomas? (in m)

A classroom contained an equal number of boys and girls. Eight girls left to play hockey, leaving twice as many boys as girls in the classsroom. What was the original number of students present?

Joe gives Nick and Tom as many peanuts as each already has. Then Nick gives Joe and Tom as many peanuts as each of them then has. Finally, Tom gives Nick and Joe as many peanuts as each has. If at the end each has sixteen peanuts, how many peanuts did each have at the beginning?